E 2212 (Pages : 3) Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2011

Fifth Semester

Core Course—MATHEMATICAL ANALYSIS

(Common for Model I and Model–II B.Sc. Mathematics and B.Sc. Computer Applications)

Time : Three Hours Maximum Weight : 25

Part A (Objective Type Questions)

Answer all *the questions.*
Each bunch of 4 *questions has weght* 1.

I. 1 Find the infimum and supremum of the set $\left\{\frac{1}{n}-\frac{1}{m}; n, m \in \mathbb{N}\right\}$

2 State Archimedean property of real numbers.

3 Prove or disprove : Every infinite set has a limit point.

4 Give an example set which is not perfect.

II. 5 Write a dense subset of $\mathbb{R}$.

6 Prove or disprove : Super set of a countable set is countable.

7 Define limit point of a set.

8 Give an example of a set which is not an interval but its derived set is [0,1].

III. 9 If possible give an example of a bounded sequence which does not converges.

10 If possible give an example of a sequence of real numbers (a_n) such that $\underline{\lim}\ a_n = \overline{\lim}\ a_n$.

11 Give an example of a Cauchy sequence.

12 Prove or disprove : A subsequence of a sequence converges then the original sequences converges.

IV. 13 Give an example of a sequence whose terms are positive but its limit is not positive.

14 Give an example of a sequence which is not monotonic but it converges to a point.

15 Write the relation between $\overline{z}^4$ and $\overline{z}^4$.

16 If $\text{Re}\, z_1 > 0$ and $\text{Re}\, z_2 > 0$ then write the relation connecting $\text{Arg}\,(z_1, z_2)$, $\text{Arg}\, z_1$ and $\text{Arg}\, z_2$.

$(4 \times 1 = 4)$

Turn over

Part B (Short Answer Questions)

Answer any five *questions.*
Each question has weight 1.

17. If $S \subseteq T \subseteq \mathbb{R}$, where $S \neq \phi$, then show that if T is bounded above, then sup $S \leq$ sup T.
18. If a be a positive real number and b any real number then show that there exists a positive integer n such that n $a > b$.
19. Show that the interior of the set N of natural numbers is the null set.
20. Show that boundedness is not necessary in order for an infinite set S to have a limit point.
21. Show that every monotonic decreasing sequence which is not bounded below, diverges to $-\infty$.
22. Show that $\lim_{n\to\infty} \frac{1}{n}\left[1+2^{1/n}+3^{1/n}+\ldots+n^{1/n}\right]=1$.
23. Show that every Cauchy sequence is bounded.
24. Prove that z is real iff $\overline{z}=z$.

$(5 \times 1 = 5)$

Part C (Short Essay Questions)

Answer any four *questions.*
Each question has weight 2.

25. Show that every open interval] a, b [contains infinitely many rational numbers.
26. Show that the interior of a set S contains every open subset of S.
27. Show that the intersection of an arbitrary family of closed sets is closed.
28. Show that every bounded sequence with a unique limit point is convergent.
29. State and prove Cauchy's second theorem on limits.
30. Let S be the open set consisting of all points z such that $|z|<1$ or $|z-2|<1$. State why S is not connected.

$(4 \times 2 = 8)$

Part D (Essay questions)

Answer any two *questions.*
Each question has weight 4.

31. Show that the derived set s' of a bounded infinite set $S\ (\subseteq \mathbb{R})$ has the smallest and the greatest members.

32. Show that the sequence $\{r^n\}$ converges iff $-1 < r \leq 1$.

33. If $\{a_n\}, \{b_n\}$ be two sequences such that $\lim_{n\to\infty} a_n = a$, $\lim_{n\to\infty} b_n = b$ then show that

    (i) $\lim (a_n b_n) = (\lim a_n)(\lim b_n) = ab$.

    (ii) $\lim \left(\frac{a_n}{b_n}\right) = \frac{\lim a_n}{\lim b_n}$, if $b \neq 0, b_n \neq 0\ \forall_n$.

$(2 \times 4 = 8)$